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STATISTICS OF NARROWBAND WHITE NOISE DERIVED FROM CLIPPED BROADBAND WHITE NOISE

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ADMINISTRATIVE INFORMATION

This work is a byproduct of a long-term analysis of broadband countermeasure signals. It is not identified with any particular project or program, but addresses a question of general concern in this area.

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13. ABSTRACT *(Maximum 200 words)*

Broadband white Gaussian noise is modelled as a series of sine and cosine functions of discrete frequencies up to a maximum frequency. The series coefficients are Gaussian with uniform variance across the frequency band. For a given sample, the Nyquist-spaced values are generated for a full period of the lowest frequency sinusoid and a peak factor is imposed, clipping the values. The resulting signal is discrete-Fourier transformed via the fast Fourier transform, hard filtered at a prescribed bandwidth, and then inverse transformed to give a set of values for the narrow band signal sample, and these values are used to update a histogram. When a sufficient number of signal samples are generated, the resulting distribution is fitted to a Gaussian distribution by a least-squares technique. It is found that the resulting distributions are noticeably non-Gaussian down to a bandwidth ratio of 1:4, below which the distribution appears to be Gaussian with a reduced variance depending on the peak factor. For a bandwidth ratio of 1:40, it is found that peak factors as low as 1.7 or even 1.5 have little effect on the narrowband distribution.

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INTRODUCTION

In many countermeasure applications, it is necessary to broadcast broadband white noise to influence a device that has a relatively narrowband processor of unknown center frequency. Due to practical limitations, the excursions of the power supply generating the broadband noise are hard limited by a peak factor. An important operational question then is: to what extent does this limit the excursions of a narrowband-filtered output of the power supply?

In this note, the broadband white noise is modelled as a finite sum of discrete-frequency sinusoids, with Gaussian-distributed amplitudes derived from a uniform distribution. The resulting time-domain signal then is hard limited, and the discrete Fourier transform is applied (exact for this type of signal) and executed via the fast Fourier transform. The resulting transform amplitudes then are hard filtered for a particular choice of bandwidth, and the inverse discrete Fourier transform is applied. Time samples of the resulting narrowband time signal then are used to update a histogram of values, the amplitudes of the broadband time signal then are re-seeded, and the process repeated until a sufficient number of signal samples are collected.

The resulting histogram is used to construct a least-squares fit to a Gaussian distribution, and the resulting variance is compared to that for the unclipped case.

THE SIGNAL MODEL

The broadband signal has the form

$$B(n\Delta t) = \sum_{m=1}^{N/2} [a_m \cos(2\pi nm\Delta t\Delta f) + b_m \sin(2\pi nm\Delta t\Delta f)] \tag{1}$$

where $\Delta f = 1/T$ and $\Delta t = 1/2F$ with T the period of the lowest frequency sinusoid, and F the highest frequency present. We will choose N to be a power of 2, and note that $1 \le n \le N$, $1 \le m \le N/2$, and $\Delta t\Delta f = 1/N$.

The coefficients are chosen using the following algorithm:

$$U_1 \in U[0,1] \quad , \quad U_2 \in U[0,1] \tag{2}$$

$$V_1 = 2U_1 - 1 \quad , \quad V_2 = 2U_2 - 1 \tag{3}$$

that is, U_1 and U_2 are chosen from a distribution uniform on the interval $[0,1]$, and are used to generate a distribution uniform on $[-1,1]$, of which V_1 and V_2 are samples(most computers generate the distribution uniform on $[0,1]$ by means of a pseudo-random number generator).

Define $S = V_1^2 + V_2^2$. If $S > 1$ then select new samples U_1 and U_2 from the uniform distribution and repeat the process. The resulting variable S is distributed uniformly on $[0,1]$. Next, define

$$X_1 = V_1\sqrt{\frac{-2\sigma^2 \ln S}{S}} \tag{4}$$

and

$$X_2 = V_2\sqrt{\frac{-2\sigma^2 \ln S}{S}}. \tag{5}$$

The resulting variables X_1 and X_2 have Gaussian distributions with zero mean and variance σ.

For a given broadband signal sample, a pseudo-random sequence is seeded, and the coefficients a_m and b_m are selected using the above procedure for $1 \le m \le N/2$. When a new signal sample is constructed, the sequence is re-seeded.

When a peak factor P is specified, the clipped signal has the form

$$C(n\Delta t) = B(n\Delta t)\,, B < \sigma P\;, C(n\Delta t) = \sigma P \text{ otherwise.} \tag{6}$$

The discrete Fourier transform of the clipped signal is given by

$$D(l\Delta f) = \sum_{n=1}^{N} C(n\Delta t)e^{-2\pi i l n/N} \tag{7}$$

with the inverse transform given by

$$C(n\Delta t) = \frac{1}{N}\sum_{l=1}^{N} D(l\Delta f)e^{2\pi i l n/N}. \tag{8}$$

The validity of this transform pair can be established by means of the identity

$$\sum_{m=1}^{N} e^{2\pi i(n-k)m/N} = \sum_{l=-\infty}^{\infty} \delta_{n,k+lN}. \tag{9}$$

NARROWBAND STATISTICS

The discrete Fourier transform and inverse transform can be executed via the fast Fourier transform algorithm. A subroutine for performing this algorithm is included as part of the code given in Appendix B. The code is written in Hewlett-Packard's Rocky Mountain Basic Version 3.0. The main body of code constructs a histogram for a particular bandwidth and peak factor choice. For a particular broadband signal sample, the time signal is transformed, and then hard filtered by eliminating the transform coefficients outside the chosen band. In doing this, it is important to observe the redundancy and symmetry of the transform coefficients. In particular, because we are using a real time series as input, the transform coefficients satisfy

$$Re\{D(l\Delta f)\} = Re\{D([N-l]\Delta f)\} \quad , \quad l = 1,\ldots,N/2 \tag{10}$$

and

$$Im\{D(l\Delta f)\} = -Im\{D([N-l]\Delta f)\} \quad , \quad l = 1,\ldots,N/2. \tag{11}$$

Consequently, a set of bandpass indices is selected symmetrically about $l = N/2$. Once the filtering is accomplished, the coefficients are inverse transformed to produce the associated time signal. Each time sample is examined and assigned to the appropriate histogram bin. Once this is done, the process is repeated until a sufficient number of samples is accumulated.

For explicit analysis, we have chosen $F = 2048$, and $\Delta f = 4$, so $N = 1024$, $T = 1/4$ and $\Delta t = 1/4096$. The broadband signal is chosen to have $\sigma = 1$, so the amplitudes a_m and b_m are chosen via the above procedure using a σ of $\sqrt{1/512}$. For the filter we consider bandwidth ratios of 1:40, 1:4, 1:2, 3:4, 7:8, 125:128, and 1:1, centered on the frequency 1024. The histogram is constructed as an array of $2M$ elements with element M corresponding to zero amplitude. In some cases, we have used $M = 100$ and in some cases $M = 1000$. It doesn't appear to make much difference in the resulting least-squares Gaussian fit. The matrix index change corresponding to σ for the unclipped case is arbitrarily chosen to be $M/10$ and this number is multiplied by the signal value and divided by the square root of the bandwidth ratio and the result added to M to give the appropriate histogram index. This will give the same distribution for the unclipped case for all bandwidth ratios. Then the effects of the peak factor will appear as departures from a Gaussian distribution, and modifications of the heights and widths of the Gaussian fits.

The distributions and Gaussian fits for a bandwidth ratio of 1:40, for peak factors of 1.7, 1.5, 1.25, 1.0, and 0.5 are shown in Figure 1. Here, and in Figure 2, the abscissa is labelled in units of σ for the unclipped case. The smooth curves are produced by the least-squares fitting procedure described in Appendix A. It is clear from the comparisons given in 1(b)-1(f) that the distributions are Gaussian. The superposition of fits in 1(a) shows that the effect of increasing the peak factor is to reduce the variance and increase the distribution's peak amplitude. It is clear from the curves in 1(a) that imposing a peak factor of 1.7, or even 1.5 has very little practical effect on the resulting narrowband distribution. Note that for peak factors of 1.5, 1.25, and 1.0, we have used $M = 1000$, and have 203 samples, whereas for peak factors of 1.7 and 0.5, we use $M = 100$, and have 146 samples.

To determine when the peak factor does have a significant impact on the shape of the distribution, we start at the other extreme, the distribution for the full bandwidth clipped signal, and reduce the bandwidth in slight increments. In Figure 2, we show the distributions, for peak factor 1.7, for bandwidth ratios 1:1, 125:128, 7:8, 3:4, 1:2, and 1:4. In the 1:1 case shown in 2(a), as expected, the distribution has a delta function behavior at $\pm 1.7\sigma$. In the next case, that for the slight reduction to a 125:128 ratio, shown in 2(b), the distribution is still strongly multimodal. The distribution is still noticeably non-Gaussian for subsequent bandwidth reductions, shown in 2(c)-2(e), but for a ratio of 1:4, shown in 2(f), the distribution is nearly Gaussian with a variance of 92 percent of that of the unclipped case. In these examples, the number of samples varies as follows: 1:1(303), 125:128(177), 7:8(178), 3:4(178), 1:2(223), and 1:4(123).

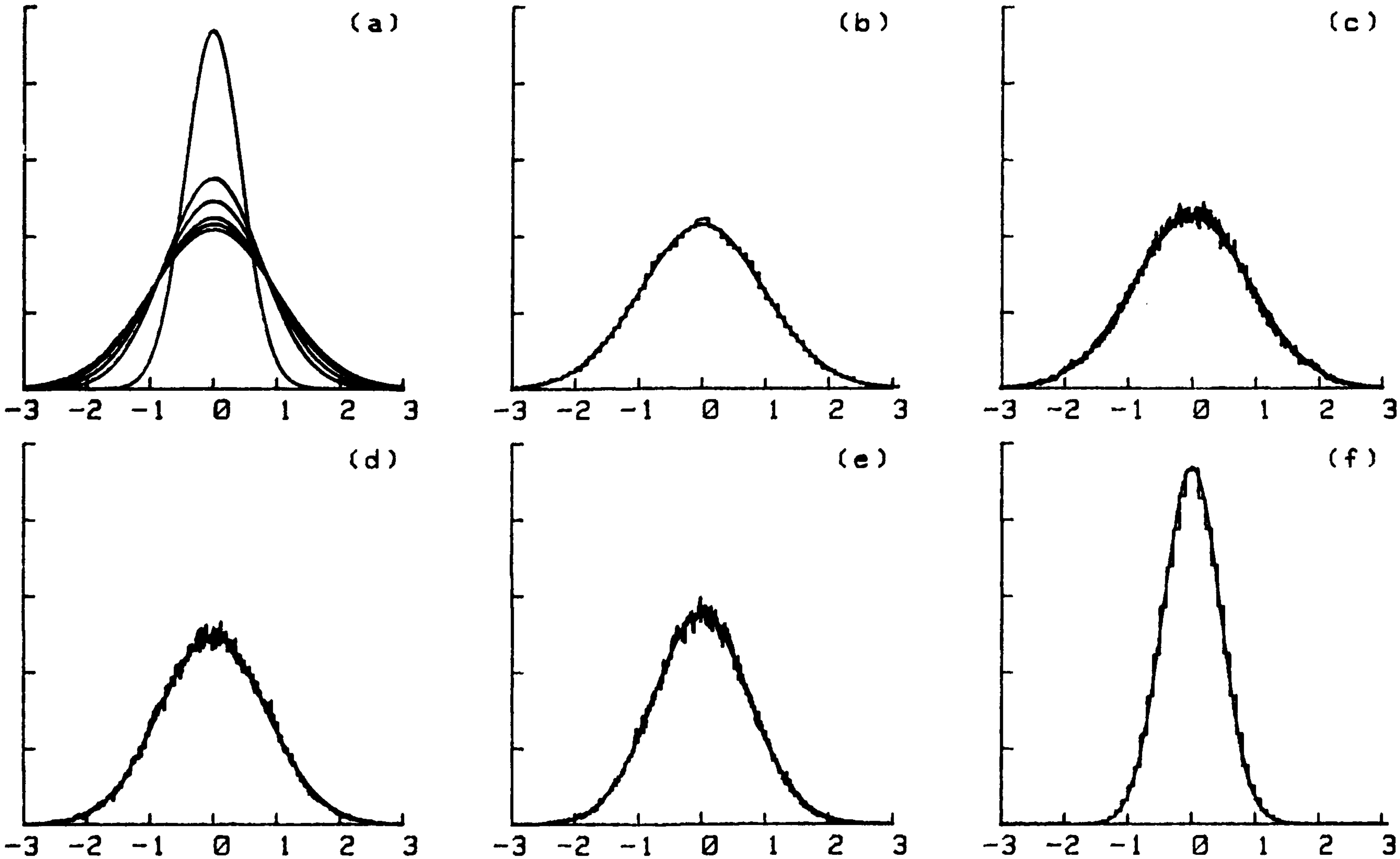

Figure 1. Gaussian Fits for a Bandwidth Ratio of 1:40, for Peak Factors of 1.7, 1.5, 1.25, 1.0 and 0.5. The Percentage Variances Listed after the Peak Factors are Given Relative to an Un-Clipped Value of 1. (a) Gaussian Fits Compared, Including the Unclipped Case. (b) 1.7, 96.8%. (c) 1.5, 93.1%. (d) 1.25, 84.9%. (e) 1.0, 75.6%. (f) 0.5, 44.5%.

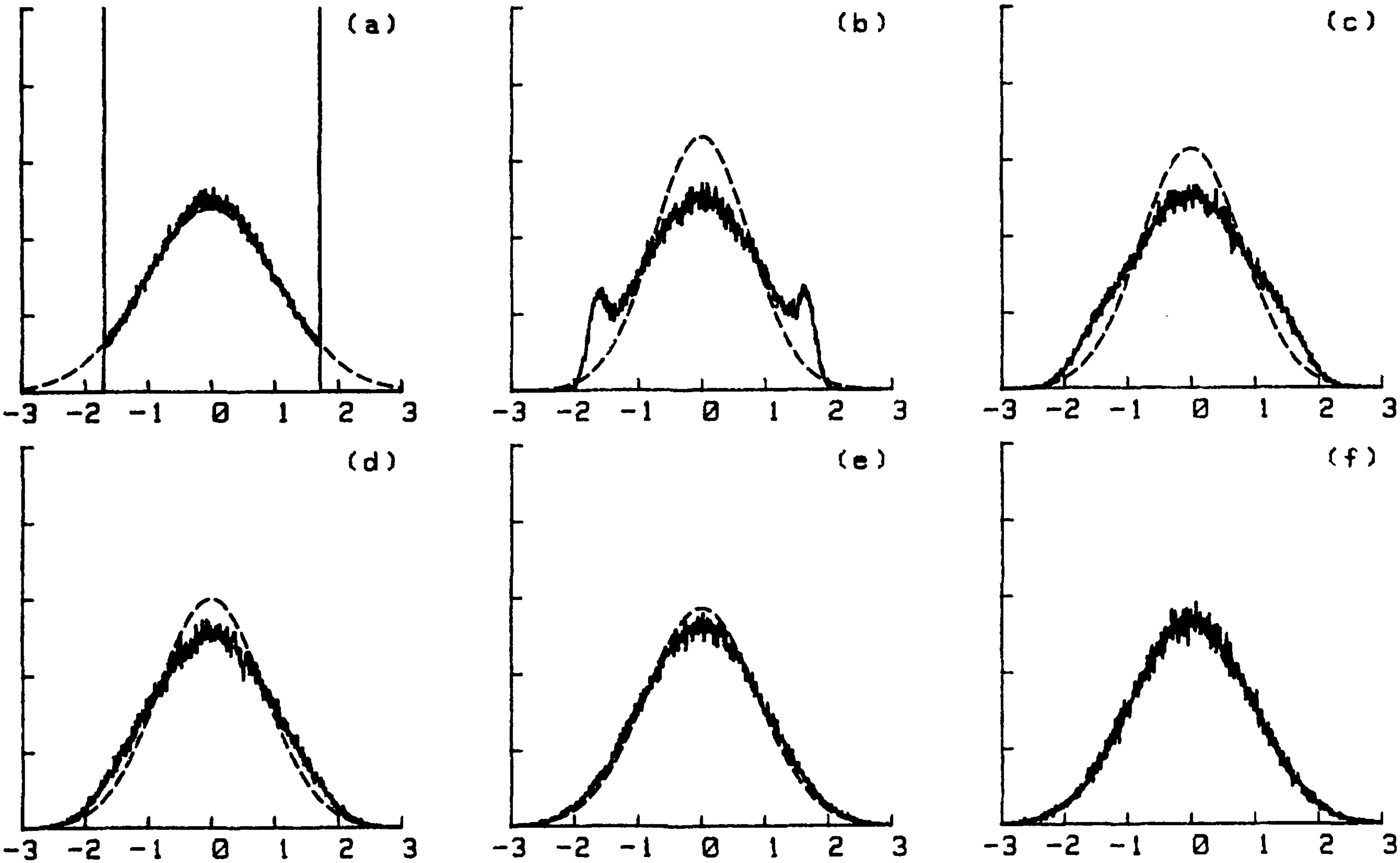

Figure 2. Histograms and Gaussian Fits Compared for a Peak Factor of 1.7, for Various Bandwidth Ratios: (a) 1:1, (b) 125:128, (c) 7:8, (d) 3:4, (e) 1:2, (f) 1:4. The Distribution is Strongly Non-Gaussian Initially, but Approaches a Gaussian Distribution of Reduced Variance as Bandwidth Decreases.

APPENDIX A

LEAST-SQUARES FITTING OF A GAUSSIAN DISTRIBUTION TO A HISTOGRAM

Given a series of variable and probability values $\{x_i, y_i\}$ where $i = 1, \ldots, N$, we would like to establish the parameter σ for the representation

$$y_i = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x_i^2}{2\sigma^2}}. \qquad (A1)$$

By taking the natural logarithm of both sides of (Eq. $A1$), we can produce the equivalent equation

$$u_i = v_i w + \ln w \qquad (A2)$$

where

$$u_i = 2\ln(\sqrt{2\pi}\, y_i) \qquad (A3)$$

$$v_i = -x_i^2 \qquad (A4)$$

and

$$w = \frac{1}{\sigma^2}. \qquad (A5)$$

The least-square fit is accomplished by numerically solving

$$\frac{d}{dw}(u_i - v_i w - \ln w)^2 = 0 \qquad (A6)$$

or, expanding, and incorporating all of the data by averaging,

$$< v_i^2 > w + < v_i > \ln w - \frac{< u_i >}{w} + \frac{\ln w}{w} = < u_i v_i > - < v_i > . \qquad (A8)$$

A subroutine for performing this process is given on the following page.

Rocky Mountain Basic 3.0 Code

```
10  SUB Gaushisfit(X(*),Y(*),Npts,S1,S2,Nmax,S)
20    !
30    !Gaussian fit to y(x) using Npts points to produce variance S
40    !S1 and S2 bound the search interval for S, Nmax is the number
50    !of partition refinements allowed.
60    !
70   OPTION BASE 1
80   DIM U(2000),V(2000)
90   REDIM U(Npts),V(Npts)
100  Lc=LOG(2*PI)
110  Nav=Npts
120  FOR I=1 TO Npts
130    IF ABS(Y(I))<1.E-10 THEN
140      U(I)=0
150      V(I)=0
160      Nav=Nav-1
170    ELSE
180      U(I)=Lc+2*LOG(Y(I))
190      V(I)=-X(I)*X(I)
200    END IF
210  NEXT I
220  A=DOT(V,V)/Nav
230  B=SUM(V)/Nav
240  C=SUM(U)/Nav
250  D=DOT(U,V)/Nav-B
260  Xl=1/S2/S2
270  Xu=1/S1/S1
280  N=0
290  Y0=A*Xl+B*LOG(Xl)-C/Xl+LOG(Xl)/Xl-D
300  Dx=(Xu-Xl)/10
310  FOR Xx=Xl TO Xu+Dx/2 STEP Dx
320    Yy=A*Xx+B*LOG(Xx)-C/Xx+LOG(Xx)/Xx-D
330    IF Yy*Y0<0 THEN
340      N=N+1
350      IF N<Nmax THEN
360        Xl=Xx-Dx
370        Xu=Xx
380        GOTO 290
390      ELSE
400        S=1/SQR(Xx)
410        GOTO 450
420      END IF
430    END IF
440  NEXT Xx
450 SUBEND
```

APPENDIX B

ROCKY MOUNTAIN BASIC 3.0 CODE FOR SIGNAL SAMPLE GENERATION, FILTERING, AND HISTOGRAM FORMATION

```
10  OPTION BASE 1
21  DIM X1(512),X2(512),C(512),S(512),Br(1024),Bi(1024),H(2000)
30  MAT H= (0)
40  Samp=0
50  INPUT "INPUT PEAK FACTOR",P
51   !Peak value is P times the broadband rms value
60  INPUT "FREQUENCY INDEX FOR NARROW BAND FILTER",Ifil
61   !Midpoint of the digital filter, In units of 4 hertz
70  INPUT "READ IN A PRIOR HISTOGRAM(Y/N)?",Ph$
80  IF Ph$="Y" THEN
90   INPUT "MSUS OF INPUT FILE?",Mh$
100   INPUT "FILE NAME?",H$
110   ASSIGN @Filein TO H$&Mh$
120   ENTER @Filein;P,Samp,H(*)
130   ASSIGN @Filein TO *
140   INPUT "DOES FILE EXIST ON MSI(Y/N)?",Fe$
150   IF Fe$="Y" THEN
160     GOTO 240
170   ELSE
180     GOTO 230
190   END IF
200  ELSE
210   INPUT "NAME OF OUTPUT FILE FOR HISTOGRAM?",H$
220  END IF
230  CREATE BDAT H$,2002,8
240  ASSIGN @File TO H$
241  INPUT "INPUT THE HALF WIDTH OF THE FILTER IN UNITS OF 4 hertz",Dlf
242   !Number of lines retained on each side of the midpoint, Ifil
243  Sig2=1/512
250   !Sigma squared for 4 hertz interval if 1 for 2048 hertz
251  RANDOMIZE
260   !Re-seed for each broadband signal sample
270  FOR I=1 TO 512
280   U1=RND
290   U2=RND
300   V1=2*U1-1
310   V2=2*U2-1
320   S0=V1*V1+V2*V2
330   IF S0>=1 THEN GOTO 280
340   Rt=SQR(-2*Sig2*LOG(S0)/S0)
350   X1(I)=V1*Rt
360   X2(I)=V2*Rt
370  NEXT I
380  Samp=Samp+1
390  PRINT Samp
400 Timesig: !
410  FOR It=1 TO 1024
411   T=(It-1)/4096
420     !Nyquist spacing for 2048 maximum frequency
430   FOR I=1 TO 512
440     C(I)=COS(2*PI*4*I*T)
450     S(I)=SIN(2*PI*4*I*T)
451       !Line spacing is 4 hertz
```

```
460    NEXT I
470    Bi(It)=0
480    Br(It)=DOT(X1,C)+DOT(X2,S)
490    IF ABS(Br(It))>P THEN
500      Br(It)=SGN(Br(It))*P
501        !Clip the signal sample
510    END IF
520  NEXT It
530  CALL Fft(Br(*),Bi(*),1024,10,-1)
531    !Get discrete transform coefficients
540  MAT Br= (1/1024)*Br
550  MAT Bi= (1/1024)*Bi
560  FOR I=1 TO 513-Ifil-Dlf-1
570    Br(I)=0
580    Bi(I)=0
590  NEXT I
600  FOR I=513-Ifil+Dlf+1 TO 513+Ifil-Dlf-1
610    Br(I)=0
620    Bi(I)=0
630  NEXT I
640  FOR I=513+Ifil+Dlf+1 TO 1024
650    Br(I)=0
660    Bi(I)=0
670  NEXT I
671    !Bandpass filter
680  CALL Fft(Br(*),Bi(*),1024,10,-1)
681    !Create the set of time values for this narrowband signal sample
690  FOR I=1 TO 1024
700    Ix=INT(Br(I)*100*SQR(2048/(2*4*Dlf))+1000)
701      !Normalize such that the histogram is independent of the bandwidth
702      !for the unclipped case
710    IF Ix>0 THEN
720      IF Ix<2001 THEN
730        H(Ix)=H(Ix)+1
731          !Upgrade the histogram
740      END IF
750    END IF
760  NEXT I
770  OUTPUT @File;P,Samp,H(*)
771    !Upgrade the stored histogram
780  GOTO 240
781    !Go back and re-seed and do it all again
790  END
800  SUB Fft(Xreal(*),Ximag(*),INTEGER N,Nu,Isign)
810    INTEGER N2,Nu1,K,K1,L,I,K1n2,P
820    N2=N DIV 2
830    Nu1=Nu-1
840    K=0
850    FOR L=1 TO Nu
860      FOR I=1 TO N2
870        CALL Bitrev(P,K DIV INT((2^Nu1)+.001),Nu)
880        Arg=2*PI*P/N
890        C=COS(Arg)
```

```
900       S=SIN(Arg)
910       K1=K+1
920       K1n2=K1+N2
930       Treal=Xreal(K1n2)*C+Ximag(K1n2)*S
940       Timag=Ximag(K1n2)*C-Xreal(K1n2)*S
950       Xreal(K1n2)=Xreal(K1)-Treal
960       Ximag(K1n2)=Ximag(K1)-Timag
970       Xreal(K1)=Xreal(K1)+Treal
980       Ximag(K1)=Ximag(K1)+Timag
990       K=K+1
1000     NEXT I
1010     K=K+N2
1020     IF K<N THEN GOTO 860
1030     K=0
1040     Nu1=Nu1-1
1050     N2=N2 DIV 2
1060   NEXT L
1070   FOR K=1 TO N
1080     CALL Bitrev(I,K-1,Nu)
1090     I=I+1
1100     IF I<=K THEN
1110       Treal=Xreal(K)
1120       Timag=Ximag(K)
1130       Xreal(K)=Xreal(I)
1140       Ximag(K)=Ximag(I)
1150       Xreal(I)=Treal
1160       Ximag(I)=Timag
1170     END IF
1180   NEXT K
1190   IF Isign=1 THEN
1200     FOR K=1 TO N
1210       Ximag(K)=-Ximag(K)
1220     NEXT K
1230   END IF
1240 SUBEND
1250 SUB Bitrev(INTEGER Ibitr,J,Nu)
1260   INTEGER I,J1,J2
1270   J1=J
1280   Ibitr=0
1290   FOR I=1 TO Nu
1300     J2=J1 DIV 2
1310     Ibitr=Ibitr*2+(J1-2*J2)
1320     J1=J2
1330   NEXT I
1340 SUBEND
```

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